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ON THE DYNAMIC INSTABILITY OF COMPONENTS IN COMPLEX STRUCTURES†

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Abstract—The case where one structural member of a larger complex structure experiences a dynamic instability is addressed. An averaged integral representation of the overall structure is chosen, except for the structural member of interest which is modelled precisely by means of the conventional methods of vibration theory. Material damping and the energy dissipation between components are taken into account by means of a distributed elastic–plastic rheological model. Particular attention is paid to keeping the intrinsic nonlinear properties of the energy dissipation. Structural members prove to act as dynamic absorbers with respect to the primary structure. The phenomenon of vibration saturation in complex structures is revealed and discussed. Full-scale vibration testing of a complex structure driven by an electromagnetic shaker is considered as an application of the concept. The moving coil of an electromagnetic shaker is shown to be dynamically unstable if an experiment is not properly designed. The influence of the large vibrating structure on the stability chart of the shaker turns out to be considerable and it changes the shaker's stability chart drastically.

1. INTRODUCTION

Typical modern complex structures are rather complicated in their composition and are actually assemblages of substructures. The attachment of the structural components to one another or to carrier surfaces results in the formation of a structure which is essentially heterogeneous in its rigidity and mass characteristics. It is difficult or even impossible to analyse the vibrations of such a structure by using conventional methods from vibration theory. Firstly, the existence of many inherently uncontrolled factors plays a principle role, see the comprehensive review papers by Ibrahim (1987) and Fahy (1994). In structural dynamics, uncertainties arise from stiffness, mass and damping fluctuations caused by variations in material properties as well as variations resulting from manufacturing and assembly. The latter gives rise to some vagueness of the boundary conditions for each structural member. Secondly, secondary systems of complex engineering structures must be included in the modelling. Thirdly, even if it were possible to obtain an "exact" boundary-value problem for a complex structure and solve it, the very interpretation of this "exact" result would present a great difficulty.

If the structure proves to be so complicated that the traditional methods of vibration theory are inadequate, it is reasonable to employ some alternative approaches. A number of integral methods have been reported, e.g. by Palmov (1979) and Belyaev and Palmov (1986). As shown in these papers, one succeeds at obtaining a rather simple boundary problem similar to the problem of dynamic visco-elasticity. The properties of the overall structure are reflected in the integral theories in the form of average rigidity and average mass as well as generalized spectra. This enables us to obtain some generalized characteristics of the vibration field and to avoid dealing with details of the structure. This level of description can be considered sufficient for many cases. However, it is not suitable for the analysis of the stability of individual components. Such analysis cannot be performed by integral methods alone, because individual components are not represented in the dynamical model. Thus, to analyse the dynamic behaviour of a component, one must take into account both its individuality and the conditions of its interactions with other components. To this end, the particular element is to be described precisely, whereas the

†Dedicated to Professor E. Brommundt, TU Braunschweig, Germany, on the occasion of his 65th birthday.

remainder of the structure is described in an integral form. This combination of overall and local modelling of vibration in weakly coupled structures, known as the local principle in structural dynamics, has been proposed by Belyaev and Palmov (1984).

This approach is applied to analyse the stability of structural members in a complex structure. It is customary to analyse the stability of a mechanical component alone and to ignore its mechanical environment, cf. the recent monograph by Leung (1993) on the dynamic analysis of substructures. The exceptions are aeroelasticity and fluid-structure interaction. However, even in these theories the interaction of a structural member with other structural members is ignored and only the interaction with fluids or gases is considered.

In the present investigation we restrict ourselves to the mutual interaction of components in a complex structure. The main intent of the present study is twofold: (i) to reveal the regions of dynamical stability of a component in a complex structure and to analyse the influence of the complex structure on the dynamic stability of the component, and (ii) to estimate the vibration of the environment when a structural member is unstable.

The paper is organised as follows. In Part 2 we derive the boundary problem which describes the local vibration in a complex structure. In Part 3 the material damping and the energy dissipation between the components of the structure are modelled by means of the generalised Prandtl rheological model. Particular attention is paid to keeping the intrinsic nonlinear properties of the energy dissipation. In Part 4 a time-reduced governing equation for the complex structure is derived. The principle regularities and peculiarities of vibration propagation in complex structures are discussed in Part 5. Part 6 deals with the full-scale vibration testing of an extended complex structure excited by an electrodynamic shaker. The regions of dynamical stability for the moving coil of an electrodynamic shaker are obtained in Part 7. The moving coil is shown to be dynamically unstable if an experiment is not properly designed. The influence of the tested structure on the stability chart of the shaker's moving coil is discussed therein. Finally, the conclusions are drawn in Part 8.

2. LOCAL VIBRATION IN COMPLEX STRUCTURE

The aforementioned combination of integral and local descriptions, namely the local principle in structural dynamics, has been reported by Belyaev and Palmov (1984, 1986). Examples of the application of this principle have been discussed by Belyaev (1990).

Let the complex structure under consideration, say, a spacecraft, be a body extending along the x axis. If longitudinal waves predominate, the structure can be represented as a one-dimensional heterogeneous rod. Any large complex engineering structure is in fact an assemblage of substructures attached to one another or to some framework. The sizes of the substructures L_n ($n = 1, 2, \ldots, N$) are much smaller than those of the entire structure L. The structural members are attached to one another or to the primary structure only at several points, so that such structures have a very low global dynamical rigidity and possess only the first, the second and seldom the third global resonances. Beyond the region of these few global resonances the vibrations localise within each substructure. This phenomenon is known as strong vibration localisation or normal mode localisation. Detailed surveys of the mode localisation phenomena in structures can be found, e.g. in the review papers by Hodges and Woodhouse (1986), Ibrahim (1987) and Li and Benaroya (1992). The studies by Cornwell and Bendiksen (1989), Pierre and Dowell (1989), Pierre (1990), Cha and Pierre (1991), Lust *et al.* (1993) and Xie and Ariaratnam (1994) are also worth mentioning. As shown in these papers, the phenomenon of mode localisation occurs not only for disordered assemblies of weakly coupled subsystems, but also for nearly periodic structures like bladed-disk assemblies and truss structures.

Consider a typical substructure L_n, see Fig. 1. Due to the vibration localisation within the substructures, the absolute displacement $v_n(x, t)$ within L_n can be sought in the form of an expansion in terms of the substructure's normal modes $v_{nk}(x)$, as follows,

$$x \in L_n \quad v_n(x, t) = \sum_{k=1}^{r} v_{nk}(x) q_{nk}(t) + V(x, t), \tag{1}$$

where $q_{nk}(t)$ is the generalised coordinate. The addition of V in eqn (1) allows one to single

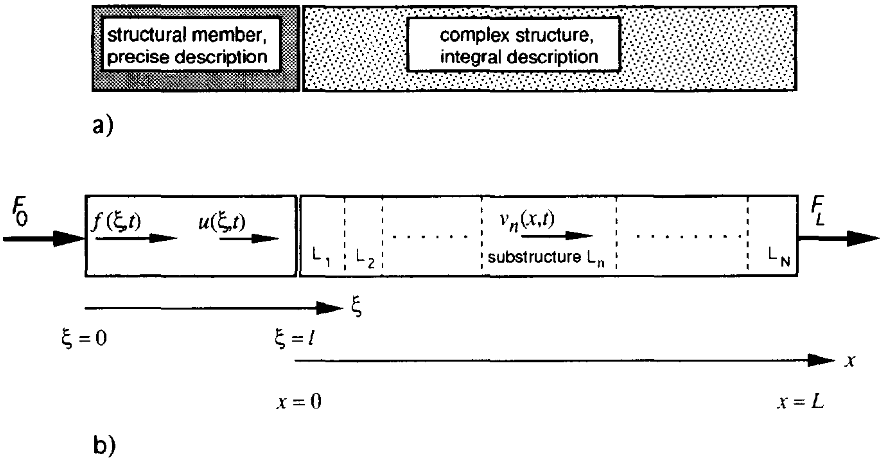

Fig. 1. (a) Combination of local and integral descriptions; (b) corresponding mechanical model.

out some mean vibrational field $V(x, t)$ that highlights the basic tendency of the vibration propagation in complex structures. The function $V(x, t)$ is usually introduced in problems of mathematical physics to improve the convergence in the vicinity of a boundary (Mindlin and Goodman (1950)). In structural dynamic plasticity a similar decomposition into a quasi-static and a dynamic part leads to an efficient numerical procedure using normal modes of the linear background structure (Ziegler and Irschik (1985), Fotiu *et al.* (1991) and Brunner and Irschik (1994)). The quasi-static part of the deformation represents the structural drift, i.e. an instantaneous measure for residual deformation.

Now the question of how to specify the substructures' normal modes arises. The normal modes are known to be sensitive to the boundary conditions, the latter being vague for any substructure, but fortunately for structural dynamists, any set of normal modes is known to be complete and, as shown in the papers on the substructure synthesis method by Meirovitch (1980) and Hale and Meirovitch (1980), the normal modes in the energy space of the substructures are not required to satisfy any boundary conditions in the internal boundaries of substructures. Hence, the normal modes may be chosen according to any suitable principle, for instance, for convenience of interpretation. Let us specify the normal modes so that they vanish at the cross-sections x_n separating the substructures, i.e. $v_{nk}(x_n) = 0$. The function V is assumed to be spatially smooth within the whole structure. In this case the function V may be referred to as the displacement of the framework since it coincides with the actual displacement of the substructures' boundaries.

Such structural members, as e.g. an axially loaded beam or a thin-walled element, are known to become dynamically unstable under certain harmonic axial loading (Bolotin (1964)). We model a potentially unstable component by a one-dimensional rod occupying $0 < \xi < l$, see Fig. 1.

The kinetic energy of the structure due to eqn (1) takes on the following form

$$T = \frac{1}{2}\int_0^l \mu \dot{u}^2 \,\mathrm{d}\xi + \frac{1}{2}\sum_{n=1}^{N}\int_{L_n} \mu \dot{v}_n^2 \,\mathrm{d}x =$$

$$\frac{1}{2}\int_0^l \mu \dot{u}^2 \,\mathrm{d}\xi + \frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{\infty} \dot{q}_{nk}^2 + \sum_{n=1}^{N}\sum_{k=1}^{\infty} \dot{q}_{nk}\int_{L_n} \mu \dot{V} v_{nk}\,\mathrm{d}x + \frac{1}{2}\int_0^L \mu \dot{V}^2 \,\mathrm{d}x, \quad (2)$$

where μ is the mass per unit length, the normal modes of the substructures are assumed to be orthonormal within each substructure and u is the absolute displacement in the element $0 < \xi < l$. $V(x, t)$ is a spatially smooth function as compared with the normal modes $v_{nk}(x)$ of the substructures and the density $\mu(x)$ of the essentially heterogeneous complex structure. Hence,

$$\int_0^L \mu \dot{V}^2\,\mathrm{d}x = \sum_{n=1}^{N} (\dot{V})_n^2 \int_{L_n} \mu\,\mathrm{d}x = \sum_{n=1}^{N} \langle\mu\rangle(\dot{V})_n^2 L_n = \int_0^L \langle\mu\rangle \dot{V}^2\,\mathrm{d}x; \quad \langle\mu\rangle = L^{-1}\int_0^L \mu\,\mathrm{d}x, \tag{3}$$

where $\langle\mu\rangle$ is an averaged mass per unit length. The latter equality in eqn (3) expresses the standard transition from the Riemann–Stieltjes sum to the corresponding integral which is admissible for large N. In the third summand in eqn (2), $\mu(x)$ and $v_{nk}(x)$ are rapidly changing functions of x, while $V(x)$ is smooth. By introduction of the average displacement of the centre of mass of the substructure L_n when it vibrates according to mode k,

$$\langle v_{nk}\rangle = \frac{1}{\langle\mu\rangle_n L_n}\int_{L_n} \mu v_{nk}\,\mathrm{d}x,$$

the kinetic energy (2) can be rewritten as

$$T = \frac{1}{2}\int_0^l \mu\dot{u}^2\,\mathrm{d}\xi + \frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{r}\{\dot{q}_{nk}^2 + 2\langle\mu\rangle_n L_n\langle v_{nk}\rangle\dot{q}_{nk}\dot{V}\} + \frac{1}{2}\int_0^L \langle\mu\rangle\dot{V}^2\,\mathrm{d}x. \tag{4}$$

In this expression the element $0 < \xi < l$ is described precisely and the rest of the structure $(0 < x < L)$ integrally.

The equation for the potential energy can be obtained analogously and is given by (Belyaev (1991))

$$\Pi = \frac{1}{2}\int_0^l c(u')^2\,\mathrm{d}\xi + \frac{1}{2}\sum_{n=1}^{N}\int_{L_n} EA(v_n')^2\,\mathrm{d}x =$$

$$\frac{1}{2}\int_0^l c(u')^2\,\mathrm{d}\xi + \frac{1}{2}\int_0^l \langle EA\rangle(V')^2\,\mathrm{d}x + \frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{r}\omega_{nk}^2 q_{nk}^2. \tag{5}$$

Here E is Young's modulus, A is the cross-sectional area, ω_{nk} is the kth eigenfrequency of the substructure n and c is the axial rigidity of the element $0 < \xi < l$. The work of the external loads is

$$W = \int_0^l fu\,\mathrm{d}\xi + F_0 u(0) + F_L V(L). \tag{6}$$

The resulting boundary value problem is then obtained by employing Hamilton's variational principle, which yields

$$0 < \xi < l: \quad c\frac{\partial^2 u}{\partial\xi^2} - \mu\frac{\partial^2 u}{\partial t^2} + f = 0 \tag{7}$$

$$x \in L_n: \quad \langle EA\rangle\frac{\partial^2 V}{\partial x^2} - \langle\mu\rangle\left[\ddot{V} + \sum_{k=1}^{r}\langle v_{nk}\rangle\ddot{q}_{nk}\right] = 0 \tag{8}$$

$$\ddot{q}_{nk} + \omega_{nk}^2 q_{nk} = -\langle\mu\rangle_n\langle v_{nk}\rangle\ddot{V} \tag{9}$$

$$x = L: \quad \langle EA\rangle V' = F_L(t) \tag{10}$$

$$\xi = 0: \quad cu' = -F_0(t). \tag{11}$$

The conditions describing the coupling of the element $0 < \xi < l$ and the remainder of the structure complete the boundary problem.

$$u(l) = V(0)\,; \quad cu'(l) = \langle EA \rangle V'(0) = F_c, \tag{12}$$

where the coupling force F_c is introduced for convenience.

3. GENERALISED PRANDTL MODEL FOR DYNAMIC ELASTO-PLASTICITY

To take into account the inherent material damping and the dissipation due to dry friction between the components of the structure, we use the generalised Prandtl model (Palmov (1976) and Belyaev (1993)). Its rheological model is composed of an infinite number of Prandtl elements as indicated in Fig. 2. Each Prandtl element (Ziegler (1995)), consists of an elastic element Edh in series with a Coulomb (or dry friction) damper which has a maximum allowable load $Ehdh$, where h is the dimensionless yield strength $(0 < h < \infty)$.

Historically, a distributed elastic-plastic model was first proposed by Ishlinsky (1944) for microplasticity problems. Later a similar model was applied by Iwan (1966) in the analysis of the dynamic response of hysteretic systems. A wide class of problems of vibrations in elastic-plastic solids was studied by Palmov (1976). Being a universal model, the generalised Prandtl model is valid for the description of nonlinear stress strain behaviour, which results in an amplitude-dependent internal friction. The model, obviously, can also be used to describe friction in the sliding or fretting of joints, supports or other parts of the structure during relative motion.

The following equations are valid for a Prandtl element

$$\mathrm{d}\sigma_h = E(\varepsilon - \varepsilon_h)\,\mathrm{d}h = Ehdh \operatorname{sign} \dot{\varepsilon}_h, \tag{13}$$

where $\mathrm{d}\sigma_h$ and ε_h are the stress and the deformation of the element having yield strength h. The summation over all elements gives the state equation for the generalised Prandtl model

$$\sigma = \int_0^{\infty} \mathrm{d}\sigma_h\,\mathrm{d}h = E\left[\varepsilon - \int_0^{\infty} \varepsilon_h R(h)\,\mathrm{d}h\right]; \quad \varepsilon = h \operatorname{sign} \dot{\varepsilon}_h + \varepsilon_h, \tag{14}$$

where σ and ε are the stress and the deformation of the material described by the generalised Prandtl model, respectively, $R(h)$ is the density of the yield strength distribution which has the following property:

$$\int_0^{\infty} R(h)\,\mathrm{d}h = 1.$$

In the case of harmonic vibrations with a frequency ω, the essentially nonlinear dependence sign $\dot{\varepsilon}_h$ in eqn (14) can be removed by means of equivalent linearization using

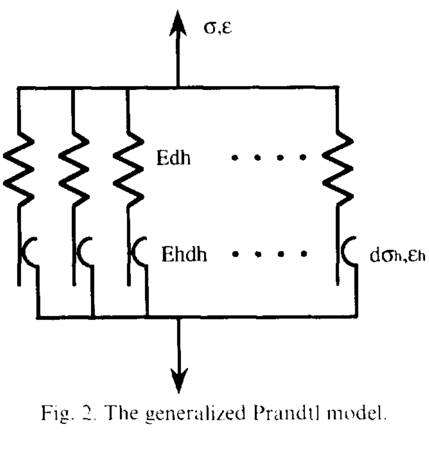

Fig. 2. The generalized Prandtl model.

the describing function method, i.e. $\operatorname{sign} \dot{\varepsilon}_h = 4[\pi\omega a_h]^{-1}\dot{\varepsilon}_h$, where a_h is the amplitude of ε_h. The second equation in (14) is then rewritten as

$$\varepsilon_h = \varepsilon[1+i4h(\pi a_h)^{-1}]^{-1} \quad \text{or} \quad a_h = \sqrt{a^2-(4\pi^{-1}h)^2}. \tag{15}$$

It is clear from the latter formula that $0 < h < 0.25\pi a$. The case $h \geqslant 0.25\pi a$ corresponds to the absence of plastic deformation, i.e. $a_h = 0$. From the first equation in (14) we have

$$\varepsilon_h = \begin{cases} \left[1-\left(\dfrac{4h}{\pi a}\right)^2 - i\dfrac{4h}{\pi a}\sqrt{1-\left(\dfrac{4h}{\pi a}\right)^2}\right]\varepsilon, & 0 < h < \dfrac{\pi}{4}a \\ 0, & h \geqslant \dfrac{\pi}{4}a \end{cases}. \tag{16}$$

Substitution of eqn (16) into eqn (14) gives

$$\sigma = E\varepsilon\left[1-\int_0^{\pi a/4}\left(1-\left(\frac{4h}{\pi a}\right)^2 - i\frac{4h}{\pi a}\sqrt{1-\left(\frac{4h}{\pi a}\right)^2}\right)R(h)\,\mathrm{d}h\right], \tag{17}$$

which finally results in the complex Young's modulus (Palmov (1976) and Belyaev (1993))

$$\sigma = \hat{E}\varepsilon; \quad \hat{E} = E\left[1-\int_0^1 (1-\eta^2 - i\eta\sqrt{1-\eta^2})R\left(\frac{\pi a\eta}{4}\right)\frac{\pi a}{4}\mathrm{d}\eta\right] = E(1+i\chi)^2. \tag{18}$$

Since $\chi(a)$ is small, the following asymptotic approximation is derived from eqn (18)

$$\chi(a) = \int_0^1 \frac{1}{2}\eta\sqrt{1-\eta^2}\,R\left(\frac{\pi a\eta}{4}\right)\frac{\pi a}{4}\,\mathrm{d}\eta. \tag{19}$$

The equation for the complex Young modulus (18) is not restricted to the case of a single harmonic. In order to investigate the cases of stationary, nonstationary, deterministic and random dynamic processes in mechanical structures, we assume that the external forces, displacements, etc., can be represented by their spectral decompositions (Ziegler (1987)),

$$V(x,t) = \int_{-\infty}^{+\infty} V(x,\omega)\mathrm{e}^{i\omega t}\,\mathrm{d}\omega; \quad F_L(t) = \int_{-\infty}^{+\infty} F_L(\omega)\mathrm{e}^{i\omega t}\,\mathrm{d}\omega; \quad q_{nk}(t) = \int_{-\infty}^{+\infty} q_{nk}(\omega)\mathrm{e}^{i\omega t}\,\mathrm{d}\omega, \tag{20}$$

etc. where $V(x,\omega)$, $F_L(\omega)$, $q_{nk}(\omega)$ are broad-band (deterministic or random) spectra. Hence the formalism offered will be valid in a wide variety of situations.

Substituting eqn (20) into eqns (7)–(12) gives the following boundary value problem in the frequency domain

$$0 < \xi < l: \quad c\frac{\partial^2 u}{\partial \xi^2} + \mu\omega^2 u + f = 0 \tag{21}$$

$$x \in L_n: \quad (1+i\chi)^2\langle EA\rangle V'' + \langle\mu\rangle\omega^2\left(V + \sum_{k=1}^{\infty}\langle v_{nk}\rangle q_{nk}\right) = 0 \tag{22}$$

$$(-\omega^2 + (1+i\chi)^2\omega_{nk}^2)q_{nk} = \omega^2\langle\mu\rangle_n\langle v_{nk}\rangle V \tag{23}$$

$$x = l: \quad (1+i\chi)^2\langle EA\rangle V' = F_L(\omega) \tag{24}$$

$$\xi = 0; \quad cu' = -F_0(\omega) \tag{25}$$

$$u(l) = V(0), \quad cu'(l) = (1+i\chi)^2 \langle EA \rangle V'(0) = F_c(\omega). \tag{26}$$

4. GOVERNING EQUATION FOR THE VIBRATION IN A COMPLEX STRUCTURE

It follows from eqn (23) that

$$q_{nk} = \frac{\omega^2 \langle \mu \rangle_n \langle v_{nk} \rangle V}{-\omega^2 + \omega_{nk}^2 (1+i\chi)^2}. \tag{27}$$

Substituting eqn (27) into eqn (22) we get the following differential equation for the vibration of the primary structure

$$(1+i\chi)^2 \langle EA \rangle V'' + \omega^2 M(\omega) V = 0, \tag{28}$$

where

$$M(\omega) = \langle \mu \rangle \left[1 + \langle \mu \rangle_n \omega^2 \sum_{k=1}^{\infty} \frac{\langle v_{nk} \rangle^2}{-\omega^2 + \omega_{nk}^2 (1+i\chi)^2} \right]. \tag{29}$$

The parameter $M(\omega)$ occupies the place of mass in the vibration equation, hence it may be called a generalised mass of the complex structure. This parameter is crucial for further analysis since it reflects the inertial and spectral properties of the complex structure. As seen from eqn (29), $M(\omega)$ is formed by an infinite number of resonance curves, each corresponding to a resonance curve of a SDOF-system. The width of each resonance curve is $2\chi\omega_{nk}$ at the "half-power" level. If the resonance curves are located so densely that

$$\Delta\omega_{nk} = |\omega_{nk+1} - \omega_{nk}| \leqslant \chi |\omega_{nk+1} + \omega_{nk}| \quad \text{or, equivalently,} \frac{\Delta\omega_{nk}}{\omega_{nk}} \leqslant 2\chi \tag{30}$$

(i.e. large modal overlap) then the resonance curves in eqn (29) merge, forming a smooth frequency function. Therefore, the sum in eqn (29) can be replaced by an integral with a locally smooth distribution function of the eigenfrequencies $\Phi(\alpha)$, i.e.

$$M(\omega) = \langle \mu \rangle \left[1 + \omega^2 \int_0^{\infty} \frac{\Phi(\alpha)\,\mathrm{d}\alpha}{-\omega^2 + \alpha^2 (1+i\chi)^2} \right]. \tag{31}$$

As seen from eqn (31), in the frequency domain of large modal overlap any complex structure has essentially a continuous spectrum of eigenfrequencies (Belyaev (1991)). Equation (31) can be now rewritten as follows

$$M(\omega) = \langle \mu \rangle [1 - i\kappa(\omega)]^2, \tag{32}$$

where κ is the non-dimensional absorption of vibration in the complex structure. We assume that the spectral properties of the complex structure are identical in all cross-sections, since they can be obtained only as a result of harmonic excitation of the entire structure.

After comparing (31) with (32), the absorption $\kappa(\omega)$ is seen to be an infinite integral

$$\kappa(\omega) = \chi\omega^2 \int_0^\infty \frac{\alpha^2\Phi(\alpha)\,\mathrm{d}\alpha}{[-\omega^2+\alpha^2(1-\chi^2)]^2+4\alpha^4\chi^2}. \tag{33}$$

By assuming that χ is small ($\chi \ll 1$) and $\Phi(\alpha)$ is smooth, the integral (33) can be determined by methods from the theory of random vibrations, Bolotin (1984), to give

$$\kappa(\omega) = \frac{\pi\omega\Phi(\omega)}{2}. \tag{34}$$

As seen from this formula, despite the vanishing material damping, the spatial absorption κ is still present and it is determined by the eigenfrequency distribution $\Phi(\alpha)$. This implies that the vibration absorption is of a resonant character and the substructures act as dynamic absorbers with respect to the framework. The dynamic properties of structures in the case of high modal overlap are analysed in detail by Belyaev (1996). It has been shown that the frequency domain of high modal overlap, known as high frequency dynamics, is a high frequency limit of vibration theory and a low frequency limit of thermodynamics.

Substituting eqn (32) into eqn (28) one obtains the governing equation for the vibration in the complex structure

$$0 < x < L: \quad \langle \hat{E}A\rangle V''+\omega^2 M(\omega)V = 0, \quad M(\omega) = \langle\mu\rangle[1-i\kappa(\omega)]^2. \tag{35}$$

5. THE PRINCIPLE REGULARITIES AND PECULIARITIES OF VIBRATION PROPAGATION IN COMPLEX STRUCTURES

The nonlinear differential eqn (35) can be solved only by means of certain asymptotic methods. Indeed, the complex Young modulus $\hat{E}$, eqn (18), depends on the deformation amplitude a, which is a derivative of $V(x)$. Let us suppose that the deformation amplitude a is a slowly varying function of x. Then the complex Young modulus $\hat{E}$ has the same property since eqn (18) for $\hat{E}$ contains an integral, which implies even further smoothing of the dependence on x. For smooth functions of x, an effective solution may be found by means of the WKB method, Heading (1962). A new variable y and a new unknown function $\Psi(y)$ are introduced as follows

$$y(x) = \int_0^x \sqrt{\frac{M(\omega)}{\langle \hat{E}(x)A\rangle}}\,\mathrm{d}x, \quad \mathrm{Im}\, y < 0; \quad V(x) = \langle \hat{E}(x)A\rangle^{-1/4}\Psi(y). \tag{36}$$

Equation (35) then takes the form

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}y^2} + \left[\omega^2 - \langle \hat{E}A\rangle^{-1/4}\frac{\mathrm{d}^2}{\mathrm{d}y^2}\langle \hat{E}A\rangle^{1/4}\right]\Psi = 0. \tag{37}$$

At the higher frequencies, the second term in the square brackets, being a second derivative of a slowly varying function, may be neglected in comparison with the first one (i.e. ω^2). Hence, the displacement $V(x)$, which asymptotically satisfies the boundary condition at the free end $x = L$, $F_L = 0$ in eqn (24), is given by

$$V(x,\omega) = B\langle \hat{E}A\rangle^{-1/4}\cos\omega[y(L)-y(x)], \tag{38}$$

where B is still to be determined from the boundary conditions (25) and (26).

Equation (38) is not the final solution, however, because the complex Young's modulus $\hat{E}$ and the new variable y depend on the deformation amplitude a and the latter is to be found through the displacement V. Therefore, at the most, eqn (38) may be considered as the equation which determines a. Obtaining a^2 asymptotically from eqn (38) yields

$$a^2(x) = \frac{1}{4}\omega^2 |B^2| \langle \hat{E}\{a(x)\} A \rangle^{-3/2} |M(\omega)| \exp\left[2\omega Im \int_x^L M(\omega)^{1/2} \langle \hat{E}\{a(x_1)\} A \rangle^{-1/2} \mathrm{d}x_1\right]. \tag{39}$$

This equation is an integral equation, since the complex Young modulus is a functional of a, $\hat{E} = \hat{E}\{a(x)\}$, (18). Taking the logarithmic derivative from a^2 yields

$$\frac{a'}{a} = \omega Im \sqrt{\frac{M(\omega)}{\langle \hat{E}\{a\} A \rangle}}. \tag{40}$$

In deriving eqn (40) we neglect terms of the form χ^2, $\chi\chi'$ etc. as asymptotically small. It obviously follows from eqn (34) that $\kappa(\omega)$ does not depend on the deformation amplitude a. Taking into account the explicit expression for $\chi(a)$ given in eqn (19), the following integrodifferential equation is obtained

$$\frac{a'}{a} = -\frac{\omega}{g}[\kappa(\omega) + \chi(a)] = -\frac{\omega}{g}\left[\kappa(\omega) + \frac{1}{2}\int_0^1 \eta\sqrt{1-\eta^2}\, R\left(\frac{\pi a \eta}{4}\right)\frac{\pi a}{4}\mathrm{d}\eta\right]; \quad g = \sqrt{\frac{\langle EA \rangle}{\langle \mu \rangle}}. \tag{41}$$

where g is the group velocity.

We borrow the distribution function $R(h)$ from the theory of internal material damping, i.e. $R(h) = \beta \tilde{R} h^{\beta-1}$ $(H > 0, \beta > 0)$ (Palmov (1976)). Evaluation of the integral in eqn (41) then renders

$$\chi(a) = \psi a^{\beta}; \quad \psi = \frac{\beta \tilde{R}}{2}\left(\frac{\pi}{4}\right)^{\beta} B\left(\frac{\beta+1}{2}; \frac{3}{2}\right), \tag{42}$$

where $B(\,;\,)$ is the Eulerian beta-function. Equation (41) may be rewritten as

$$\frac{a'}{a} = -\frac{\omega\kappa}{g} - \frac{\omega\psi}{g}a^{\beta}. \tag{43}$$

The solution of eqn (43) is given by

$$a(x) = \left[\left(a(0)^{-\beta} + \frac{\psi}{\kappa}\right)\exp\left(\frac{\beta\omega\kappa}{g}x\right) - \frac{\psi}{\kappa}\right]^{-1/\beta}. \tag{44}$$

An analysis of $a(x)$ makes it possible to determine the basic tendencies of wave propagation in complex structures. For large values of x we can neglect the last term in eqn (44), to obtain

$$a(x) \cong \left[(a(0))^{-\beta} + \frac{\psi}{\kappa}\right]^{-1/\beta} \exp\left(-\frac{\omega\kappa}{g}x\right) = a(0)\left\{1 + \frac{\chi[a(0)]}{\kappa}\right\}^{-1/\beta} \exp\left(-\frac{\omega\kappa}{g}x\right). \tag{45}$$

If the loss of mechanical energy from plastic deformations is negligible in comparison with the resonance absorption $(\chi(a) \leqslant \chi[a(0)] \ll \kappa)$ then

$$a(x) \cong a(0)\exp(-\omega\kappa x/g). \tag{46}$$

It is clear from eqn (46), that the absorption of energy is determined by the value of $\kappa(\omega)$. The latter is given through the function of the eigenfrequency distribution $\Phi(\omega)$, cf. eqn

(34). Hence, the internal degrees of freedom of each substructure correspond to a set of dynamic absorbers with respect to the carrier structure, thus providing considerable spatial absorption of the energy of propagating waves in the whole high-frequency domain.

If the condition $\chi \ll \kappa$ does not hold, i.e. if material damping, frictional and plastic effects are considerable, the structure demonstrates evident nonlinear properties. For example, for any amplitude of the external force the following inequality holds

$$a(x) < \left\{\frac{\psi}{\kappa}\left[\exp\left(\frac{\beta\omega\kappa x}{g}\right)-1\right]\right\}^{1/\beta} = a_m(x). \tag{47}$$

The latter formula indicates an upper limit of the deformation amplitude at any point of the structure, even if the power of the external source of excitation is unbounded (Belyaev and Ziegler (1991). Hence, this level does not depend on the power of the source, but is a function of the spatial distance from the source and the mechanical characteristics of the structure. This implies that for any complex structure we can plot a universal curve which is a majorant of the curves of the amplitude of the actual deformation. The existence of such an upper limit of the field of deformation means that the structure can conduct only a limited amount of vibrational energy. This phenomenon of vibration saturation in complex structures can be explained only by means of nonlinear analysis.

For a numerical example we choose the following parameters: $\omega = 2\pi \cdot 500\ \mathrm{s}^{-1}$, $g = 2 \cdot 10^3\ \mathrm{ms}^{-1}$, $\kappa = 0.2$. The yield strength h is assumed to be uniformly distributed ($\beta = 1$), hence it follows from eqn (42) that $\chi = \psi a$. We take $\psi = 5$, corresponding to the energy loss parameter $\chi = 0.05$ when the deformation amplitude is $a = 0.01$. The actual deformation $a(x)$, eqn (44) and the majorant $a_m(x)$, eqn (47), are presented in Fig. 3.

From Fig. 3 one can see that the majorant gives a rather rough estimate in the neighbourhood of the excitation. This is not surprising since the majorant is an upper bound for the deformation field, even for an arbitrarily large excitation. Further along the structure (say, for $x > 5$ m) the majorant is rather close to the actual field of deformation. This result implies that the amplitude of the external force cannot be determined by measurements from the far field of vibration. Indeed, the deformation curves and the

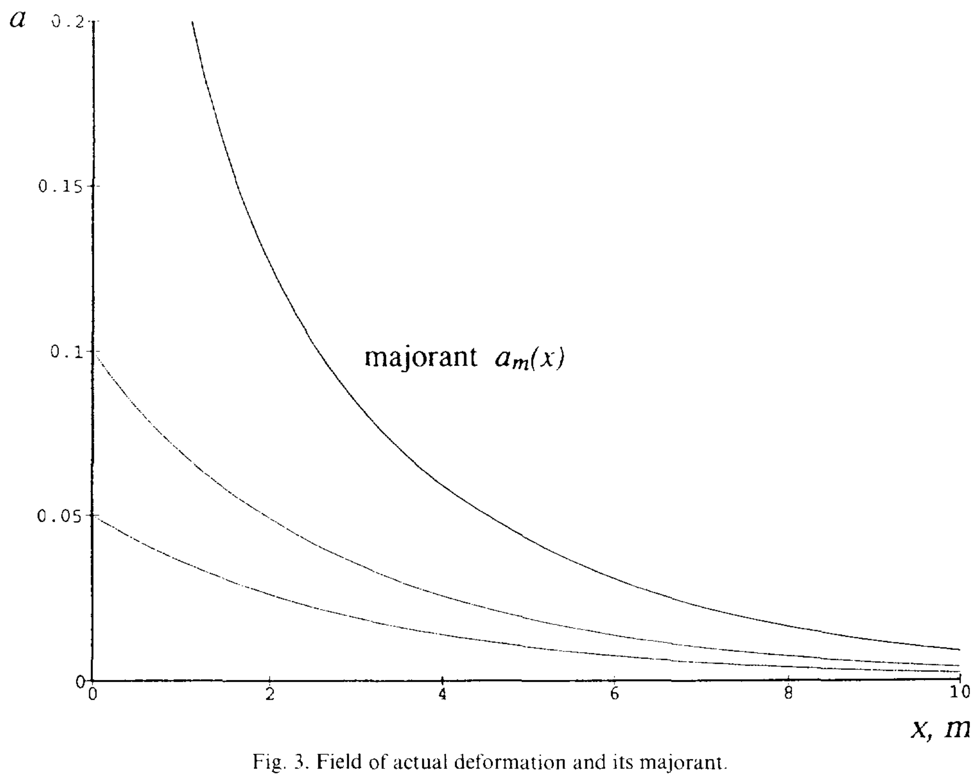

Fig. 3. Field of actual deformation and its majorant.

majorant are very close, however the majorant does not depend on the intensity of the excitation at all.

6. AN EXAMPLE: VIBRATION TESTING OF AN EXTENDED COMPLEX STRUCTURE AT HIGH FREQUENCIES

Parts 6 and 7 address the full-scale vibration testing of a spacecraft which is driven by an electrodynamic shaker. By means of this example of a practical application of the approach we intend to highlight the effect of a large vibrating structure on the stability of the moving coil of an electrodynamic shaker. In this particular example the shaker's coil plays the role of the element which becomes unstable while the spacecraft represents a typical complex structure.

High-power equipment and particularly powerful vibration exciters are required for many purposes (Ibrahim (1991)). In some cases the response of the test component is significant and it has a backward effect on the shaker armature. This backward effect can be avoided by using a shaker whose output is sufficiently strong that it is not significantly affected by the test component response.

The simulation of a vibration environment for aerospace structures and their components represents an important situation in which such shakers are required. A failure in the vibration testing of a large spacecraft was discussed by Belyaev (1994) and is briefly reviewed here. It is a generally accepted practice to incorporate into the design of spacecraft special explosive devices (e.g. explosive bolts) that function as a release mechanism to separate two subassemblies of the spacecraft (Bucciarelli and Askinazi (1973)). Mounted directly on the structure, a pyrotechnic generates a considerable dynamic environment which can affect sensitive equipment carried aboard the spacecraft, especially in the neighbourhood of the pyrotechnic. In the full-scale testing of a spacecraft, a powerful electrodynamic shaker failed to produce the same vibration level that was caused by the firing of a small explosive bolt. As explained in Part 5 of the present study, any complex structure can conduct only a limited amount of vibrational energy, and the vibration field has an upper limit $a_m(x)$. Therefore, the simulation may fail if the shaker is mounted rather far from a test component and the desired level of vibration is higher than the majorant $a_m(x)$. An excessive increasing of the shaker's driving force does not result in a proportional increase of the vibration amplitude in the structure, but it does result in a widening of the plastic zone. Also, an unbounded increase of the driving force can result in dynamic buckling of the shaker coil and consequently in its damage.

The two main components of the electrodynamic shaker are a stationary magnet and a moving coil as depicted in Fig. 4. The moving coil is in fact a spiral coil whose turns are glued and fastened by a few brace rods, hence it can be modelled by a circular cylindrical

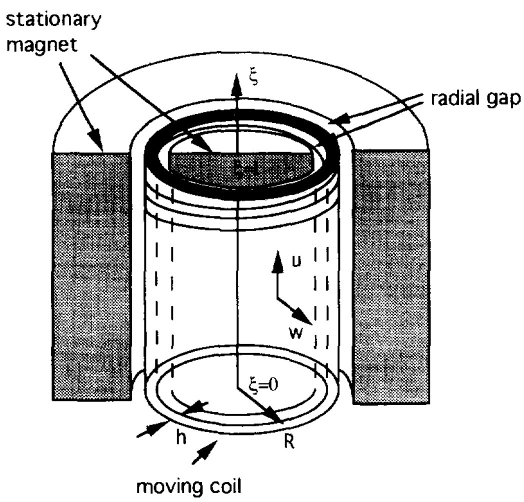

Fig. 4. Schematic of elestrodynamic shaker.

shell. For certain relations between the shell parameters and the excitation frequency, the membrane state is known to be dynamically unstable (Bolotin (1964)). The deflections of the middle surface from the initial membrane state are referred to as u and w. The circumferential displacement vanishes because of the axial symmetry of the problem. If we neglect the influence of the bending vibration on the axial one, then the longitudinal vibration of the shaker is governed by eqn (7), i.e.

$$0 < \xi < l : cu'' - \mu\ddot{u} + f\cos\omega t = 0\,; \quad u' = \frac{\partial u}{\partial \xi}; \quad \dot{u} = \frac{\partial u}{\partial t}. \tag{48}$$

Here $c = 2\pi RhE$ is the axial rigidity, E is Young's modulus, ρ is the mass density, R and h are the radius and the thickness of the shell respectively, $\mu = 2\pi Rh\rho$ is the mass density per length unit and $f\cos\omega t$ is a harmonic axial force per length unit created by the shaker's coil. The coil end $\xi = 0$ is free ($F_0 = 0$), hence

$$\xi = 0: \quad u' = 0. \tag{49}$$

Balancing the axial displacements and the axial coupling forces of the shaker and the tested structure gives

$$u(l) = V(0)\,; \quad cu'(l) = \langle \hat{E}(0)A\rangle V'(0) = F_c. \tag{50}$$

The solution of the differential eqn (48) that satisfies the boundary condition (49) is given by

$$u(\xi,\omega) = -\frac{f}{\mu\omega^2} + H\cos\frac{\omega\xi}{s}, \tag{51}$$

where $s = (E/\rho)^{1/2}$ is the velocity of sound. The unknown coefficients B in eqn (38), and H, above, are obtained from the condition of the coupling of the coil and the complex structure, eqn (50), yielding

$$B = \frac{-f\langle \hat{E}(0)A\rangle^{1/4}\sin\dfrac{\omega l}{s}}{\mu\omega^2\cos[\omega y(L)]\left\{\sin\dfrac{\omega l}{s} + \dfrac{s\langle \hat{E}(0)A\rangle^{1/2}M^{1/2}\tan[\omega y(L)]}{c}\cos\dfrac{\omega l}{s}\right\}} \tag{52}$$

$$H = \frac{f\tan[\omega y(L)]}{\mu\omega^2\left\{\tan[\omega y(L)]\cos\dfrac{\omega l}{s} + \dfrac{c}{s\langle \hat{E}(0)A\rangle^{1/2}M^{1/2}}\sin\dfrac{\omega l}{s}\right\}}. \tag{53}$$

The following estimation is asymptotically valid

$$\tan[\omega y(L)] = \frac{1}{i}\frac{\exp[i\omega y(L)] - \exp[-i\omega y(L)]}{\exp[i\omega y(L)] + \exp[-i\omega y(L)]} = -i \tag{54}$$

for extended complex structures at high frequencies ($-Im\omega y(L) \gg 1$). This helps to simplify eqns (52) and (53) and to obtain the following expressions for u and V

$$V(x,\omega) = -\frac{f}{\mu\omega^2}\frac{\langle \hat{E}(0)A\rangle^{1/4}}{\langle \hat{E}(x)A\rangle^{1/4}}\frac{\sin\dfrac{\omega l}{s}}{\sin\dfrac{\omega l}{s}-\dfrac{is\langle \hat{E}(0)A\rangle^{1/2}M^{1/2}}{c}\cos\dfrac{\omega l}{s}}\frac{\cos\omega[y(L)-y(x)]}{\cos[\omega y(L)]} \tag{55}$$

$$u(\xi,\omega) = \frac{f}{\mu\omega^2}\left[-1+\frac{\cos\dfrac{\omega\xi}{s}}{\left\{\cos\dfrac{\omega l}{s}+\dfrac{ic}{s\langle \hat{E}(0)A\rangle^{1/2}M^{1/2}}\sin\dfrac{\omega l}{s}\right\}}\right]. \tag{56}$$

7. REGIONS OF DYNAMICAL STABILITY OF A MOVING COIL OF AN ELECTRODYNAMIC SHAKER

When studying dynamic stability it is customary to analyse the equation in variations (Bolotin (1964)). In our case the equation in variations for flexural vibration of a circular cylindrical shell (Timoshenko and Woinowsky-Krieger (1959) and Bolotin (1964)), is given by

$$w^{\mathrm{IV}}+\frac{Eh}{DR^2}w+\frac{12\nu}{h^2R}u'+\frac{1}{D}\left(\frac{N}{2\pi R}w'\right)'+\frac{\rho h}{D}\ddot{w}=0;\quad '=\frac{\partial}{\partial\xi}, \tag{57}$$

where ν is Poisson's ratio, D is the flexural rigidity and $N = cu'$ is the axial elastic force. Substituting the axial displacement

$$u(\xi,t) = u(\xi,\omega)\exp(i\omega t) = \left[-\frac{f}{\mu\omega^2}+H\cos\frac{\omega\xi}{s}\right]\exp(i\omega t) \tag{58}$$

into the equation of bending vibration of the coil, eqn (57), yields

$$w^{\mathrm{IV}}+\frac{Eh}{DR^2}w-H\frac{12\nu\omega}{h^2Rs}\sin\frac{\omega\xi}{s}\exp(i\omega t)-\frac{\mu s\omega H}{2\pi RD}\left(\sin\frac{\omega\xi}{s}w'\right)'\exp(i\omega t)+\frac{\rho h}{D}\ddot{w}=0. \tag{59}$$

Equation (59) has the same form as the equation describing beam buckling under parametric excitation. Each elemental strip of the shell acts like a beam on an elastic Winkler foundation, the role of this foundation being played by the circumferential forces. Now we formulate the boundary conditions for eqn (59). The end $\xi = 0$ is free, i.e.

$$\xi = 0:\quad w'' = 0,\ w''' = 0. \tag{60}$$

The coil's end $\xi = l$ is usually attached to the armature which is in fact a very rigid structural member, hence one can write

$$\xi = l:\quad w = 0,\ w' = 0. \tag{61}$$

The Galerkin method is used to solve the boundary problem, eqns (59)–(61). The Galerkin procedure usually demands a large number of terms. However, for the stability analysis the series may be truncated to a single term if the fundamental function $\varphi(\xi)$ is chosen properly, i.e. $\varphi(\xi)$ is the form of the dynamic buckling shape. Therefore, the solution is sought in the following form

$$w(\xi, t) = \varphi(\xi)q(t)\,; \quad q(t) = \int_0^l w(\xi, t)\varphi(\xi)\,\mathrm{d}\xi\,; \quad \int_0^l \varphi^2(\xi)\,\mathrm{d}\xi = 1, \tag{62}$$

where the fundamental function $\varphi(\xi)$ must satisfy the boundary conditions (60) and (61).

In order to obtain the equation for the generalised coordinate $q(t)$ we multiply eqn (59) by $\varphi(\xi)$ and integrate along the length of the shell. The result is

$$\ddot{q}+\frac{D}{\rho h}\left[\int_0^l \varphi^{\mathrm{IV}}\varphi\,\mathrm{d}\xi+\frac{Eh}{DR^2}-\frac{\mu s\omega H\exp(i\omega t)}{2\pi RD}\int_0^l\left[\varphi'\sin\frac{\omega}{s}\xi\right]'\varphi\,\mathrm{d}\xi\right]q = -\frac{12\nu DH\omega\exp(i\omega t)}{Rh^3 s\rho}\int_0^l \varphi\sin\frac{\omega\xi}{s}\mathrm{d}\xi. \tag{63}$$

Equation (63) can be rewritten in a form analogous to the Mathieu equation

$$\ddot{q}+\Omega^2\left[1-\frac{2f}{f_c}\exp(i\omega t+\alpha)\right]q = Q\exp(i\omega t+\alpha)\,; \quad Q = -\frac{12\nu P\omega D|H|}{s\rho Rh^3}\int_0^l \varphi\sin\frac{\omega\xi}{s}\mathrm{d}\xi, \tag{64}$$

where $\alpha = \arg H$. The natural frequency of the bending vibration of the axially loaded shell Ω and the critical load per unit length f_c are given by

$$\Omega^2 = \frac{D}{\rho h}\left[\int_0^l \varphi^{\mathrm{IV}}\varphi\,\mathrm{d}\xi+\frac{Eh}{DR^2}\right] \tag{65}$$

$$f_c = \frac{4\pi\omega\rho hR\Omega^2}{s}\left|\frac{\cos\dfrac{\omega l}{s}+i\sqrt{\dfrac{c\mu}{\langle\hat{E}(0)A\rangle\langle M\rangle}}\sin\dfrac{\omega l}{s}}{\displaystyle\int_0^l(\varphi')^2\sin\frac{\omega l}{s}\mathrm{d}\xi}\right|, \tag{66}$$

where the denominator in eqn (66) has been simplified by means of integration by parts.

An approximate solution of eqn (64) is sought in the following form (Bolotin (1964) and Roseau (1987))

$$q(t) = p_2\sin(\omega t+\alpha)+p_1\sin([\omega t+\alpha]/2)+q_0+q_1\cos([\omega t+\alpha]/2)+q_2\cos(\omega t+\alpha). \tag{67}$$

Substituting eqn (67) into eqn (64) and equating the coefficients of identical exponential functions gives the amplitudes of the driven vibration of the coil

$$p_2 = 0\,; \quad q_0 = \frac{\dfrac{f}{f_c}Q}{-\dfrac{\omega^2}{4}+\Omega^2\left(1-2\dfrac{f^2}{f_c^2}\right)}\,; \quad q_2 = \frac{Q}{-\dfrac{\omega^2}{4}+\Omega^2\left(1-2\dfrac{f^2}{f_c^2}\right)} \tag{68}$$

and the equation of the boundaries of the regions of dynamical instability of the coil

$$\omega = 2\Omega\sqrt{1 \pm \frac{f}{f_c}}. \tag{69}$$

Equation (69) is known to be the first approximation to the boundaries of the principal region of instability (Bolotin (1964) and Roseau (1987)), so formally the stability boundaries coincide with the customary boundaries of the Mathieu equation. The principal difference is that the critical load f_c depends on the excitation frequency ω and the complex Young modulus $\hat{E}$, cf. (66). The latter however is a function of ω and the amplitude of deformation $a(0)$ which in turn depends on the excitation force f. Thus one should expect a stability chart that differs from the customary one.

Specifying the fundamental function $\varphi(\xi)$ allows us to obtain the values of the eigenfrequency Ω and the critical load f_c. Equations (65) and (66) yield the exact values of the eigenfrequency Ω and the critical load f_c if the chosen fundamental function $\varphi(\xi)$ is of the same form as that of the free vibration and static buckling. The normal modes and the buckling forms are known to coincide only for free supported shells (the Navier boundary conditions). Since the coil is modelled by a cantilever shell which does not satisfy these conditions, one is facing a choice. If $\varphi(\xi)$ is chosen as a normal mode, then the Galerkin method (62) gives the exact value of Ω and an approximate value of f_c. If the buckling form is taken, the exact critical force and an approximate eigenfrequency will be obtained. As shown by Bolotin (1964), for the first three regions of instability of a cantilever beam, the difference between the forms of free vibration and static buckling is so small that the discrepancies both in eigenfrequency and the Euler critical force do not exceed 1%.

Let us specify the fundamental function $\varphi(\xi)$ as the first normal mode of the axisymmetric bending vibrations of the circular cylindrical shell, i.e.

$$\varphi(\xi) = \frac{1}{\sqrt{l}}\left[\cosh\frac{\gamma(l-\xi)}{l} - \cos\frac{\gamma(l-\xi)}{l} - \theta\left(\sinh\frac{\gamma(l-\xi)}{l} - \sin\frac{\gamma(l-\xi)}{l}\right)\right], \tag{70}$$

where $\gamma = 1.875$ and $\theta = 0.734$. Substituting eqn (70) into eqns (65) and (66) one obtains the first eigenfrequency

$$\Omega^2 = \frac{D}{\rho h}\left[\left(\frac{\gamma}{l}\right)^4 + \frac{Eh}{DR^2}\right]. \tag{71}$$

In order to compute the regions of instability one needs $a(0)$. Substituting B, eqns (52) and (54), into eqn (39) and setting $x = 0$ one has

$$a(0) = \frac{cf}{\mu s\omega\langle EA\rangle(1+\chi^2)}\left|\frac{\sin\frac{\omega l}{s}}{\left\{\cos\frac{\omega l}{s} + \frac{i(c\mu)^{1/2}}{\langle EA\rangle^{1/2}\langle\mu\rangle^{1/2}(1+i\chi)(1-i\kappa)}\sin\frac{\omega l}{s}\right\}}\right|. \tag{72}$$

In fact, eqn (72) is an equation for $a(0)$, since χ is a function of a, e.g. $\chi = \psi a$ as in the numerical example in Part 5. Under the assumption $\chi = \psi a$ eqn (72) can be rewritten as

$$a(0) = \frac{cf}{\mu s\omega\langle EA\rangle(1+\psi^2 a(0)^2)}\left|\frac{\sin\frac{\omega l}{s}}{\left\{\cos\frac{\omega l}{s} + \frac{i(c\mu)^{1/2}}{\langle EA\rangle^{1/2}\langle\mu\rangle^{1/2}(1+i\psi a(0))(1-i\kappa)}\sin\frac{\omega l}{s}\right\}}\right|. \tag{73}$$

This equation for $a(0)$ can be solved only numerically. The following parameters of the shaker's moving coil and the complex structure (e.g. the example in Part 5) are taken for the numerical work: $R = l = 0.5$ m, $h = 10^{-2}$ m, $s = 5 \cdot 10^3$ m s^{-1}, $\rho = 7.8 \cdot 10^3$ kg m^{-3}, $\langle \mu \rangle = 10^3$ kg m^{-1}, $\langle EA \rangle = 10^9$ N, $g = 2 \cdot 10^3$ m s^{-1}, $\psi = 5$ and $\kappa = 0.2$. We substitute the obtained value of $a(0)$ into eqn (66) to determine the critical force f_c. The latter is substituted into eqn (69) to obtain the boundaries of the principal region of dynamical instability. Figure 5 shows these regions for the moving coil alone and for the moving coil attached to the complex structure. As seen from Fig. 5, the influence of the complex structure on the principal region of dynamical instability is considerable. It is easy to establish the condition under which one can ignore the backward effect of the tested structure's vibration on the coil's stability chart. Provided that the mechanical impedance of the coil is much smaller than that of the complex structure, i.e. $\sqrt{c\mu} \ll \sqrt{\langle EA \rangle \langle \mu \rangle}$, the principal region of dynamical instability, eqns (69), (65) and (66), becomes the conventional one of the Mathieu equation. A similar analysis of the stability chart of a shaker coil which excites a small test component (not a complex structure) was performed by Belyaev and Irschik (1995).

It is also instructive to plot the principal region of instability in the plane of coordinates $\omega/2\Omega$ and $a(0)$, Fig. 6. This figure shows that attempts to simulate considerable deformations are doomed to failure and will result in the dynamic buckling of the shaker's coil. The latter conclusion, together with the existence of the upper limit of the vibration amplitude field, allows us to explain the vibration testing failure described in Part 6.

In connection with the example considered, the following question arises: how large are the deformation amplitudes in a complex structure if one of its structural members becomes unstable? The present analysis dealt only with the boundaries of the instability region and cannot help at all in this regard. Nonlinear stability analysis is required to estimate the vibration in the complex structure when one of the structural members experiences instability. Nevertheless, some estimation can be performed. The upper bound of the amplitude field $a_m(x)$, Part 5, corresponds to the case of infinitely large force of excitation. This is exactly the case where, in the framework of the linear stability analysis, a thin-walled component experiences instability. Therefore, $a_m(x)$ represents the vibration field in the complex structure if the linear stability analysis could be applied at large deformations.

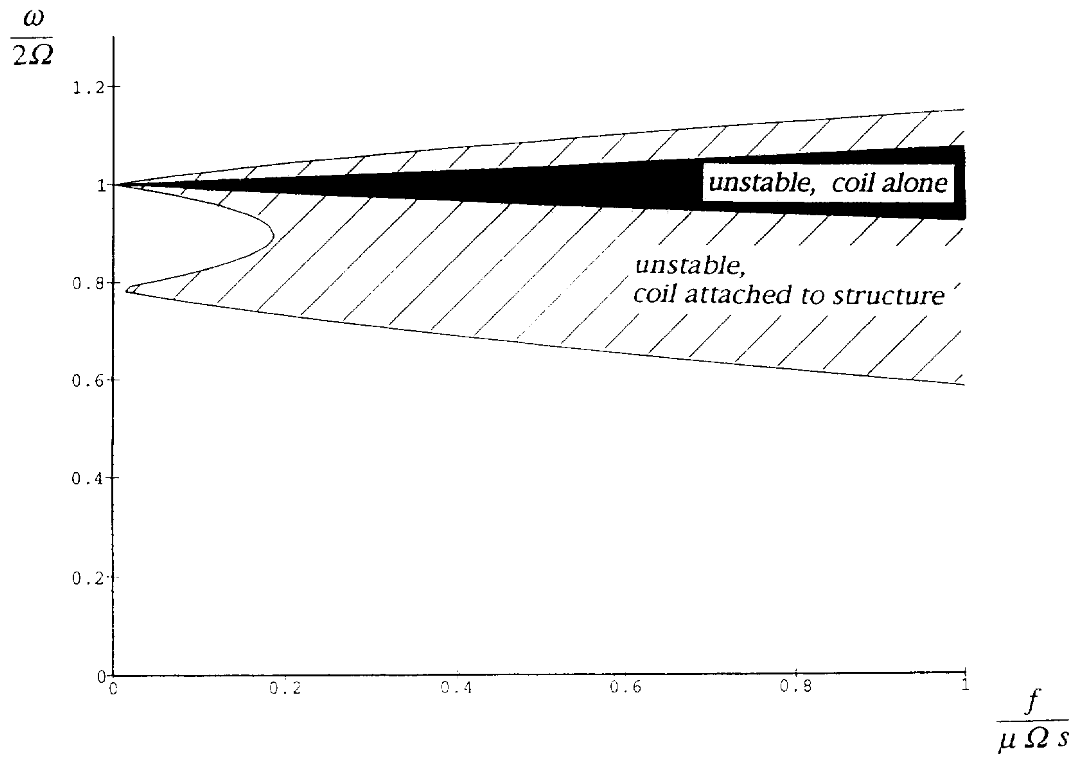

Fig. 5. Stability chart of the shaker's coil.

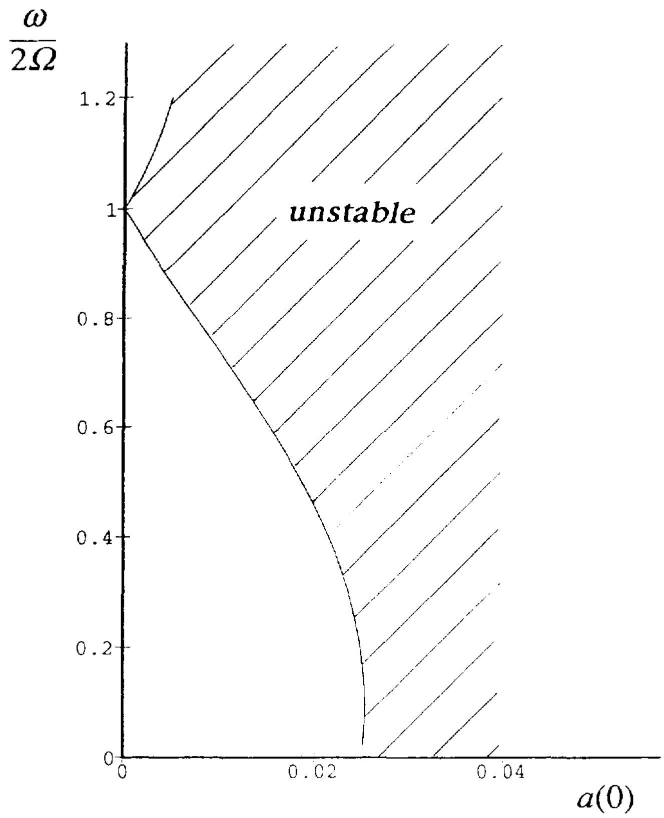

Fig. 6. Stability chart of the shaker's coil.

Since, strictly speaking, any structural member at large deformations demonstrates its nonlinear properties, the buckling amplitude is bounded, and $a_m(x)$ should be considered the upper bound of the vibration field in the complex structure, even when one structural member becomes unstable.

8. CONCLUSIONS

The dynamical instability of components in complex structures was addressed. Because of the great computational difficulties stemming from the pedantic descriptions of all details and peculiarities of actual structures, another strategy was chosen. A potentially unstable component was described precisely whereas the remainder of the complex structure was described by means of an integral method. The approach was applied to determine the vibration field in the complex structure and the vibrations of an individual structural member.

The present study was in fact limited by obtaining the vibration of the primary structure. An account of the internal degrees of freedom was made by means of a generalised mass of the complex structure. The secondary structures prove to act as the dynamic absorbers for the primary structure, providing resonant absorption of vibrational energy. A second source of the considerable energy dissipation is the material damping and dry damping between structural members during their relative motion. The nonlinear nature of these types of dissipation causes various nonlinear phenomena. The yield strength distribution function chosen in the present investigation allows us to describe the effect of vibration saturation in complex structures. Another interesting nonlinear effect has been reported by Belyaev *et al.* (1996). For another distribution function, a maximum distance in which a disturbance propagates has been revealed. If this maximum distance is exceeded, wave motion in the structure is completely arrested.

As an example of the application of the present approach, the vibration testing of an extended structure was considered. Similar to any thin-walled axially loaded element, a moving coil of an electrodynamic shaker can be dynamically unstable. The backward effects of a large vibrating structure on the regions of dynamical instability of the coil turn out to be considerable, and it increases the instability region considerably. Although not addressed here, a study of stochastic stability of structural members in actual structures can also be developed based on this concept.

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